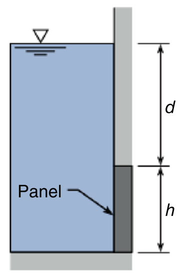
3.68: PROBLEM DEFINITION

Situation:

Water exerts a load on square panel.
$d = 1\,\mathrm{m}$, $h = 2\,\mathrm{m}$



Find:

(a) Depth of the centroid (m).
(b) Resultant force on the panel (kN).
(c) Distance from the centroid to the center of pressure (m).

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\mathrm{N/\,m^3}$.

PLAN

1. Locate the centroid by inspection (center of the panel).
2. Find the pressure at the depth of the centroid using the hydrostatic equation.
3. Find the resultant force using $F = \overline{p}A$.
4. Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$

SOLUTION

1. Depth of the centroid of area:

$$\begin{aligned} \overline{z} &= d + h/2 = 1\,\mathrm{m} + (2\,\mathrm{m})/2 \\ & \boxed{\overline{z} = 2\,\mathrm{m}} \end{aligned}$$

2. Hydrostatic equation:

$$\overline{p} = \gamma \overline{z} = \left(9800\,\mathrm{N/\,m^3}\right)(2\,\mathrm{m}) = 19.6\,\mathrm{kPa}$$

3. Resultant force:

$$\begin{aligned} F &= \overline{p}A = (19.6\,\mathrm{kPa})(2\,\mathrm{m})(2\,\mathrm{m}) \\ & \boxed{F = 78.4\ \mathrm{kN}} \end{aligned}$$

4. Distance to CP:

- Find $\overline{I}$ using formula from Fig. A.1.

$$\overline{I} = \frac{bh^3}{12} = \frac{(2\,\mathrm{m})\,(2\,\mathrm{m})^3}{12} = 1.333\,\mathrm{m}^4$$

- Recognize that $\bar{y} = \overline{z} = 2\,\mathrm{m}$.
- Final calculation:

$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{(1.333\,\mathrm{m}^4)}{(2\,\mathrm{m})\,(2\,\mathrm{m})^2}$$

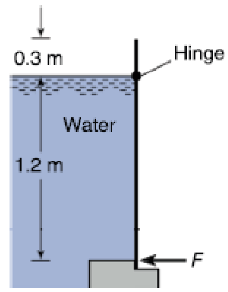
$$\boxed{y_{cp} - \bar{y} = 0.167\ \mathrm{m}}$$

3.72: PROBLEM DEFINITION

Situation:

A rectangular gate is hinged at the water line.

$h = 1.2\,\text{m}$, $b =$ 1.8 m



Find:

Force to keep gate closed.

Properties:

From Table A.4, $\gamma_{\text{Water}} = 9810\ \text{N/m}^3$.

SOLUTION

Hydrostatic Force (magnitude):

$$\begin{aligned} F_G &= \bar{p}A \\ &= (\gamma_{H_2O} \times \bar{y})\ (2.16\ \text{m}^2) \\ &= (9810\,\text{N/m}^3 \times 0.6\,\text{m})\ (2.16\ \text{m}^2) \\ &= 12\ 714\ \text{N} \end{aligned}$$

Center of pressure. Since the gate extends from the free surface of the water, F_G acts at 2/3 depth or 0.8 m below the water surface.

Moment Equilibrium. (sum moments about the hinge)

$$\begin{aligned} \sum M &= 0 \\ (F_G \times 0.8\,\text{m}) - (1.2\,\text{m})\,F &= 0 \end{aligned}$$

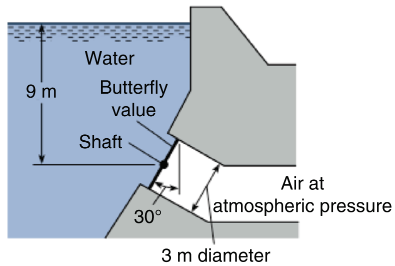
$$F = \frac{12\ 714\ \text{N x } 0.8\ \text{m}}{1.2\,\text{m}}$$

$$\boxed{F = 8\ 476\ \text{N} \quad \text{to the left}}$$

3.76: PROBLEM DEFINITION

Situation:

A butterfly valve is described in the problem statement.
$d = 3\,\text{m}$, $\theta = 30\,°$, $\bar{y} = 9\,\text{m}$.



Find:

Torque required to hold valve in position (N-m).

SOLUTION Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \bar{y}\gamma A \\ &= (9 \text{ m} \times 9810 \text{ N/m}^3)\left(\pi \times \frac{D^2}{4}\right) \text{ m}^2 \\ &= \left(9\,\text{m} \times 9810\,\text{N/ m}^3 \times \pi \times \frac{(3\,\text{m})^2}{4}\right) \\ &= 623,770 \text{ N} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\ &= \frac{\pi r^4/4}{\bar{y}\pi r^2} \\ &= \frac{(1.5\,\text{m})^2/4}{9\,\text{m}/0.866} \\ &= 0.054 \text{ m} \end{aligned}$$

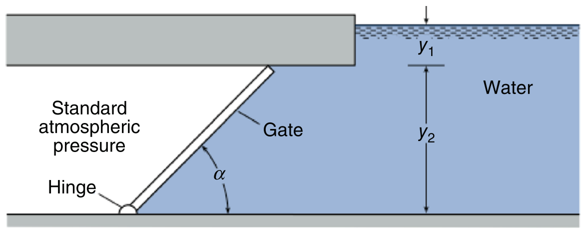
Torque

$$\begin{aligned} \text{Torque} &= 0.054\,\text{m} \times 623,770\,\text{N} \\ & \boxed{T = 33,685 \text{ N-m}} \end{aligned}$$

3.77: PROBLEM DEFINITION

Situation:

A submerged gate may fall due to its weight (or be held in place by pressure).
$y_1 = 1\,\mathrm{m}$, $y_2 = 4\,\mathrm{m}$, $w = 1\,\mathrm{m}$.
$W = 150\,\mathrm{kN}$, $\alpha = 45^o$.



Find:

Will the gate fall or stay in position?

Properties:

Water (10 °C), Table A.5, $\gamma = 9810\,\mathrm{N/\,m^3}$.

SOLUTION

1. Geometry

- Slant height:

$$\bar{y} = \frac{y_1 + y_2/2}{\sin\alpha} = \frac{(1+4/2)\,\mathrm{m}}{\sin 45^o} = 4.243\,\mathrm{m}$$

- Depth of centroid:

$$\Delta z = y_1 + \frac{y_2}{2} = \left(1 + \frac{4}{2}\right)\,\mathrm{m} = 3\,\mathrm{m}$$

- Panel surface area

$$A = \left(\frac{y_2}{\sin\alpha}\right) w = \left(\frac{4\,\mathrm{m}}{\sin 45^o}\right)(1\,\mathrm{m}) = 5.657\,\mathrm{m}^2$$

2. Pressure at Centroid:

$$\bar{p} = \gamma\Delta z = \left(9810\,\mathrm{N/\,m^3}\right)(3\,\mathrm{m}) = 29.43\,\mathrm{kPa}$$

3. Hydrostatic force:

$$F = \bar{p}A = (29.43\,\mathrm{kPa})\left(5.657\,\mathrm{m}^2\right) = 166.5\,\mathrm{kN}$$

4. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1:

$$\begin{aligned} \bar{I} &= \frac{wh^3}{12} \\ h &= \frac{y_2}{\sin\alpha} = \frac{4\,\mathrm{m}}{\sin 45^o} = 5.657\,\mathrm{m} \\ \bar{I} &= \frac{wh^3}{12} = \frac{(1\,\mathrm{m})\,(5.657\,\mathrm{m})^3}{12} = 15.09\,\mathrm{m}^4 \end{aligned}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{(15.09\,\mathrm{m}^4)}{(4.243\,\mathrm{m})\,(5.657\,\mathrm{m}^2)} = 0.6287\,\mathrm{m}$$

5. Torques:

- Torque caused by hydrostatic force:

$$x_h = \frac{h}{2} - (y_{cp} - \bar{y}) = \frac{5.657\,\mathrm{m}}{2} - 0.6287\,\mathrm{m} = 2.200\,\mathrm{m}$$

$$T_{\mathrm{HS}} = Fx_h = (166.5\,\mathrm{kN})\,(2.2\,\mathrm{m}) = 366\,\mathrm{kN}\cdot\,\mathrm{m}$$

- Torque caused by the weight:

$$x_w = \frac{y_2/2}{\tan\alpha} = \frac{4\,\mathrm{m}/2}{\tan 45^o} = 2\,\mathrm{m}$$

$$T_{\mathrm{W}} = Wx_w = (150\,\mathrm{kN})\,(2\,\mathrm{m}) = 300\,\mathrm{kN}\cdot\,\mathrm{m}$$

The torque caused by the hydrostatic force exceeds the torque caused by the weight: So the **gate will stay in position.**

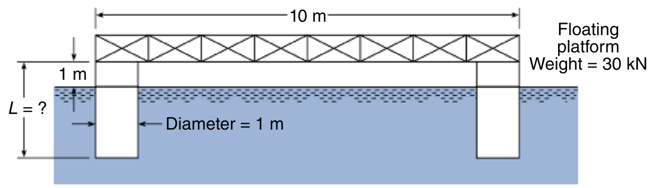
3.103: PROBLEM DEFINITION

Situation:

A platform floats in water.

$W_{\text{platform}} = 30\,\text{kN}$, $W_{\text{cylinder}} = 1\,\text{kN/m}$.

$y = 1\,\text{m}$, $D_{\text{cylinder}} = 1\,\text{m}$.



Find:

Length of cylinder so that the platform floats 1 m above water surface.

Properties:

$\gamma_{\text{water}} = 10,000\,\text{N/m}^3$.

SOLUTION

1. Equilibrium (vertical direction)

$$\begin{pmatrix}\text{Weight of}\\ \text{platform}\end{pmatrix} + 4\begin{pmatrix}\text{Weight of}\\ \text{a cylinder}\end{pmatrix} = 4\begin{pmatrix}\text{Buoyant force}\\ \text{on a cylinder}\end{pmatrix}$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\left(\gamma V_D\right)$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\left(\frac{10000\,\text{N}}{\text{m}^3}\right)\left(\frac{\pi\,(1\,\text{m})^2}{4}\,(L - 1\,\text{m})\right)$$

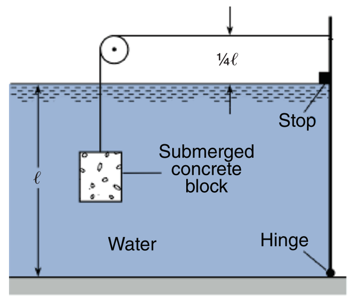
2. Solve for L

$$\boxed{L = 2.24\,\text{m}}$$

3.105: PROBLEM DEFINITION

Situation:

A submerged gate has a concrete block attached to it.
$b = 1\,\text{m}$, $\ell = 2\,\text{m}$.



Find:

Minimum volume of concrete to keep gate in closed position (m^3).

Properties:

Concrete $\gamma = 23.6\,\text{kN/m}^3$.

SOLUTION

Hydrostatic force on gate and CP

$$F = \bar{p}A = 1\,\text{m} \times 9,810\,\text{N/m}^3 \times 2\,\text{m} \times 1\,\text{m} = 19,620\ \text{N}$$

$$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{1\,\text{m} \times (2\,\text{m})^3}{12 \times 1\,\text{m} \times 2\,\text{m} \times 1\,\text{m}} = 0.33\ \text{m}$$

Sum moments about the hinge to find the tension in the cable

$$T = 19,620 \times \frac{1 - 0.33}{2.5} = 5,258\ \text{N}$$

Equilibrium applied to concrete block

$$\begin{pmatrix}\text{Tension} \\ \text{in cable}\end{pmatrix} + \begin{pmatrix}\text{Buoyant} \\ \text{force}\end{pmatrix} = (\text{Weight})$$

$$T + V\gamma_{H_2O} = V\gamma_c$$

Solve for volume of block

$$V = \frac{T}{\gamma_c - \gamma_{H_2O}} = \frac{5258\,\text{N}}{23,600\,\text{N/m}^3 - 9,810\,\text{N/m}^3}$$

$$\boxed{V = 0.381\ \text{m}^3}$$

4.5: PROBLEM DEFINITION

Situation:

Dye is injected into a flow field and produces a streakline.

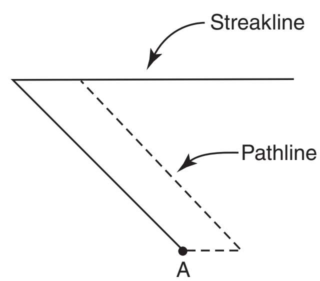
Pathline starts at $t = 4\,\mathrm{s}$, ends at $t = 10\,\mathrm{s}$. Flow speed is constant.

Find:

Draw a pathline of the particle.

SOLUTION

The streakline shows that the velocity field was originally in the horizontal direction to the right and then the flow field changed upward to the left. The pathline starts off to the right and then continues upward to the left.



4.25: PROBLEM DEFINITION

Situation:

A path line is given with velocity as a function of distance and time.
$V = s^2 t^{1/2}$, $r = 0.4\,\text{m}$.
$s = 1.5\,\text{m}$, $t = 0.5$ s.

Find:

Acceleration along and normal to pathline (m/s^2).

PLAN

Apply Eq. 4.11 of EFM 10e for acceleration along pathline.

SOLUTION

Equation 4.5

$$\mathbf{a} = (V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t})\mathbf{u}_t + \left(\frac{V^2}{r}\right)\mathbf{u}_n$$

Evaluation of velocity and derivatives at $s = 2$ m and $t = 0.5$ sec.

$$\begin{aligned} V &= s^2 t^{1/2} = 1.5^2 \times 0.5^{1/2} = 1.59 \text{ m/s} \\ \frac{\partial V}{\partial s} &= 2st^{1/2} = 2 \times 1.5 \times 0.5^{1/2} = 2.121/\text{s} \\ \frac{\partial V}{\partial t} &= \frac{1}{2}s^2 t^{-1/2} = \frac{1}{2} \times 1.5^2 \times 0.5^{-1/2} = 1.59 \text{ m/s}^2 \end{aligned}$$

Evaluation of the acceleration

$$\mathbf{a} = (1.59 \times 2.12 + 1.59)\mathbf{u}_t + \left(\frac{1.59^2}{0.4}\right)\mathbf{u}_n$$

$$\boxed{\mathbf{a} = 4.96\mathbf{u}_t + 6.32\mathbf{u}_n \;\; (\text{m/s}^2)}$$

4.44: PROBLEM DEFINITION

Situation:

Velocity varies linearly with distance in water nozzle.
$L = 0.3\,\text{m}$, $V_1 = 9\,\text{m/s}$, $V_2 = 24\,\text{m/s}$.

Find: Pressure gradient midway in the nozzle (kPa/m).

Properties:
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply Euler's equation.

SOLUTION

Euler's equation

$$\frac{\partial}{\partial x}(p + \gamma z) = -\rho a_x$$

but z =const.; therefore

$$\frac{\partial p}{\partial x} = -\rho a_x$$

$$a_x = a_{\text{convective}} = V\frac{\partial V}{\partial x}$$

$$\frac{\partial V}{\partial x} = (24 - 9)/0.3 = 50\ \text{s}^{-1}$$

$$V_{\text{mid}} = (24\,\text{m/s} + 9\,\text{m/s})/2 = 16.5\ \text{m/s}$$

$$a_x = (16.5\ \text{m/s})(50\ \text{m/s/m}) = 825\ \text{m/s}^2$$

Finally

$$\frac{\partial p}{\partial x} = (1000\ \text{kg/m}^3)(825\ \text{m/s}^2)$$

$$\boxed{\frac{\partial p}{\partial x} = -825\ \text{kPa/m}}$$

4.50: PROBLEM DEFINITION

Situation:

Water discharges from a pressurized tank.

$z_1 = 0.5\,\text{m}$, $z_2 = 0\,\text{m}$, $V_1 = 0\,\text{m/s}$.

Find:

Velocity of water at outlet (m/s).

Properties:

Water (20 °C, 10 kPa), Table A.5: $\rho = 998\,\text{kg/m}^3$, $\gamma = 9790\,\text{N/m}^3$.

SOLUTION

Apply the Bernoulli equation between the water surface in the tank (1) and the outlet (2)

$$p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}$$

Neglect V_1 ($V_1 \ll V_2$).Also $p_2 = 0$ gage. The Bernoulli equation reduces to

$$\rho \frac{V_2^2}{2} = p_1 + \gamma(z_1 - z_2)$$

$$V_2 = \sqrt{\frac{2\left(p_1 + \gamma(z_1 - z_2)\right)}{\rho}}$$

Elevation difference $z_1 - z_2 = 0.5$ m. For water at 20^{o}C, $\rho = 998\text{kg/m}^3$ and $\gamma = 9790$ N/m^3.Therefore

$$V_2 = \sqrt{\frac{2(10,000\,\text{Pa} + 9790\,\text{N/m}^3\,(0.5\,\text{m}))}{998\,\text{kg/m}^3}}$$

$$\boxed{V_2 = 5.46\,\text{m/s}}$$

4.55: PROBLEM DEFINITION

Situation:

A glass tube with 90° bend inserted into a stream of water.

$V = 5\,\text{m/s}$.

Find:

Rise in vertical leg above water surface (m).

PLAN

Apply the Bernoulli equation.

SOLUTION

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

where d is depth below surface and h is distance above water surface.

Bernoulli equation (between free stream and stagnation point)

$$\begin{aligned}\frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\ h + d &= d + \frac{V^2}{2g} \\ h &= \frac{V^2}{2g}\end{aligned}$$

$$h = \frac{(5\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)}$$

$$\boxed{h = 1.27\,\text{m}}$$

4.62: PROBLEM DEFINITION

Situation:

Two Pitot tubes are connected to air-water manometers to measure air and water velocities.

Find:

The relationship between V_A and V_W .

$$V = \sqrt{2g\Delta h} = \sqrt{\frac{2\Delta p_z}{\rho}}$$

SOLUTION

The Δp_z is the same for both; however,

$$\rho_w >> \rho_a$$

Therefore $V_A > V_W$. The correct choice is **b).**

4.81: PROBLEM DEFINITION

Situation:

A two-dimensional velocity field is given by:

$u = \frac{Cx}{(x^2+y^2)}$, $v = \frac{Cy}{(x^2+y^2)}$.

Find:

Check if flow is irrotational.

SOLUTION

Apply equations for flow rotation in $x - y$ plane.

$$\begin{aligned} \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} &= \frac{-2xCy}{(x^2+y^2)^2} - \left[-\frac{2yCx}{(x^2+y^2)^2} \right] \\ &= 0 \\ & \boxed{\text{The flow is irrotational}} \end{aligned}$$

4.101: PROBLEM DEFINITION

Situation:

A closed tank with liquid is rotated about the vertical axis.
$\omega = 10$ rad/s, $r_B = 0.5\,\text{m}$, $a_z = 4\,\text{m/s}^2$.

Find:

Difference in pressure between points A and B (kPa).

Properties:

$\rho = 1000\,\text{kg/m}^3$, $S = 1.2$.

PLAN

Apply the pressure variation equation for rotating flow between points B & C. Let point C be at the center bottom of the tank.

SOLUTION

Pressure variation equation- rotating flow

$$p_B - \frac{\rho r_B^2 \omega^2}{2} = p_C - \frac{\rho r_C^2 \omega^2}{2}$$

where $r_B = 0.5$ m, $r_C = 0$ and $\omega = 10$ rad/s. Then

$$\begin{aligned} p_B - p_C &= \frac{\rho}{2}(\omega^2)(r^2) \\ &= \frac{1200\,\text{kg/m}^3}{2}(100\,\text{rad}^2/\text{s}^2)(0.25\,\text{m}^2) \\ &= 15,000 \text{ Pa} \\ p_C - p_A &= 2\gamma + \rho a_z \ell \\ &= 2\left(11,772\,\text{N/m}^3\right) + \left(1,200\,\text{kg/m}^3\right)\left(4\,\text{m/s}^2\right)(2) \\ &= 33.1 \text{ kPa} \end{aligned}$$

Then

$$\begin{aligned} p_B - p_A &= p_B - p_C + (p_C - p_A) \\ &= 15,000\,\text{Pa} + 33,144\,\text{Pa} \\ &= 48,144\,\text{Pa} \\ & \boxed{p_B - p_A = 48.1 \text{ kPa}} \end{aligned}$$